Lecture 1.

Matrix. Rules of matrix algebra.

We say that we have a matrix A, if A is a rectangular array of elements displayed in rows and columns and enclosed in square or round brackets.

Elements of matrices may be numbers, variables, polynomials or other expressions.

Let a matrix A consists of m rows and n columns. It is a matrix of order $m \times n$. If m=n then the matrix is said to be square. Real numbers are 1×1 matrices. A vector (x,y) in the plane is a 1×2 matrix. This matrix is said to be of order 1×2 . So, a vector (x,y,z) in the space is said to be a 1×3 matrix. Real number, when used in matrix computations, is called scalar.

Rules of matrix algebra.

If the matrices A and B have the same size, then their sum is the matrix A+B defined by (A + B)ij = aij + bij . Their difference is the matrix A − B defined by (A − B)ij = aij − bij .

A matrix A can be multiplied by a scalar c to obtain the matrix cA, where (cA)ij = caij . This is called scalar multiplication. We just multiply each entry of A by c.

The m × n matrix whose entries are all 0 is denoted 0mn (or, more often, just by 0 if the dimensions are obvious from context). It’s called the zero matrix.

Two matrices A and B are equal if all their corresponding entries are equal: A = B ⇐⇒ aij = bij for all i, j.

If the number of columns of A equals the number of rows of B, then the product AB is defined. If AB is defined, then the number of rows of AB is the same as the number of rows of A, and the number of columns is the same as the number of columns of B: $A_{m\times n} \cdot B_{n\times l} = (AB)_{m\times l}$

If both matrices are square and of the same size, so that both AB and BA are defined and have the same size, the two products are not generally equal.

A is square if it has the same number of rows and columns. An important instance is the identity matrix I, which has ones on the main diagonal and zeros elsewhere. The identity matrices behave, in some sense, like the number 1. If A is n × m, then IA = A, and AI = A.

Suppose A and B are square matrices of the same dimension, and suppose that AB = I = BA. Then B is said to be the inverse of A